The ANR Impossibility for Rankings as a Corollary of Abstract Tie-Breaking

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Abstract

We give a new proof that no anonymous, neutral, resolute aggregation rule produces a complete ranking of $m \geq 3$ alternatives when $\gcd(n, m!) > 1$, where n is the number of voters. The result itself is known. The impossibility direction was established by Moulin (1983) for single-winner selection under a Pareto assumption, and extended without Pareto to ranking output by Bubboloni and Gori (2014). What is new is the proof. It is a single application of the impossibility theorem (Theorem A, i.e., Theorem 13) of the abstract tie-breaking framework developed in Feys (2026). No voting-specific technique is used. We show that $\gcd(n, m!) > 1$ is exactly the condition under which symmetric anonymous profiles exist in the framework's sense, and we apply the characterization and decomposition theorems of Feys (2026) to describe the forced structure when ties are permitted: the orbit partition Ω records what the profile determines, and the completion records what it does not. The contribution is not the theorem but what the proof reveals, namely that the ANR impossibility is not a voting-specific phenomenon but a group-theoretic consequence of $\mathrm{Sym}(\mathcal{N})$-equivariance and the absence of fixed points in the space of linear orders.

1 Introduction

1.1 The Problem

For three alternatives and six voters, no voting rule can simultaneously treat all voters alike, treat all alternatives alike, and always produce a definitive ranking. The obstruction is arithmetic: $\gcd(6, 3!) = 6 \neq 1$. In fact, the impossibility holds for most parameter combinations. For $m = 3$ alternatives, a fair resolute ranking rule exists only when the number of voters is coprime to 6, that is, for $n = 1, 5, 7, 11, 13, \ldots$. For $m = 4$, the condition tightens to coprimality with 24. This is the ANR impossibility (anonymity, neutrality, resoluteness), a classical theme in social choice theory whose impossibility direction was established by Moulin (1983) for single-winner selection under a Pareto assumption, and extended without Pareto to ranking output by Bubboloni and Gori (2014).

In the present note, we show that the impossibility direction for rankings is a direct corollary of the abstract tie-breaking framework developed in Feys (2026), a framework

designed for non-voting settings (chess tournaments, sports leagues, cooperative games, network centrality). The proof consists of verifying that anonymous profiles constitute an instance of the framework, exhibiting a symmetric profile, and applying the framework's impossibility theorem. The contribution is not the result, which is known, but what the proof reveals: the ANR impossibility is driven by the same group-theoretic obstruction that makes strict tie-breaking impossible in these other settings.

1.2 The State of the Literature

Three lines of work in the ANR tradition are relevant. Moulin (1983) proved the foundational result: an anonymous, neutral, efficient, and resolute social choice function on n voters and m alternatives exists if and only if $\gcd(n, m!) = 1$. Bubboloni and Gori (2014, 2021) extended the analysis to ranking output and to weakened axioms determined by subgroups, establishing the same arithmetic condition without the Pareto assumption. Özkes and Sanver (2021) revisited the ANR impossibility with a focus on consequential neutrality as a weakening that restores resoluteness. Xia (2023) introduced "most equitable refinements," using the group-theoretic observation that a nontrivial permutation cannot fix a strict ranking to define "fixed-point decisions" and to construct rules that optimally preserve anonymity and neutrality at non-problematic profiles. For a comprehensive survey of the broader field, see Brandt et al. (2016).

Each of these works operates within the voting-specific framework of profiles, voter permutations, and alternative permutations. The present note takes a different route.

1.3 The Methodological Move

We do not reprove the ANR impossibility from scratch. Instead, we observe that it is a direct corollary of Theorem A of Feys (2026) (numbered as Theorem 13 in the body of that work), the impossibility theorem of the abstract tie-breaking framework. That theorem was proved in a setting with no voters, no ballots, and no social choice apparatus. For the reader's convenience, we recall the result informally. Theorem A states that if a $\mathrm{Sym}(\mathcal{N})$-set $\mathcal{I}$ contains a *symmetric* element, i.e., an element whose stabilizer includes a nontrivial permutation, then no $\mathrm{Sym}(\mathcal{N})$-equivariant function $\mathcal{I} \to \mathcal{L}(\mathcal{N})$ exists. The ANR impossibility falls out as a special case once one verifies that anonymous profiles form such a $\mathrm{Sym}(\mathcal{N})$-set and that symmetric profiles exist when $\gcd(n, m!) > 1$.

1.4 Contributions

Three specific contributions follow.

The arithmetic equivalence (Proposition 3.1). The condition $\gcd(n, m!) > 1$ is exactly the condition under which symmetric anonymous profiles exist in the framework's sense. This connects the number-theoretic condition of the ANR tradition to the abstract notion of symmetric input from Feys (2026).

The corollary proof (Theorem 4.1). The ANR impossibility for rankings follows from Theorem A in three lines. The result is known from Bubboloni and Gori (2014). What is new is that the proof uses no voting-specific technique.

The orbit partition on anonymous profiles (Section 5). When ties are permitted, the characterization theorem (Theorem B of Feys 2026) identifies the orbit partition Ω as the unique honest partition-valued rule, and the decomposition theorem (Theorem C) factors every partition-consistent strict rule as Ω followed by an arbitrary completion. We work out what this structure looks like concretely on anonymous profiles.

1.5 Scope

Our contribution is the proof method and the conceptual interpretation it provides, not the result itself. The result is also weaker than Arrow's impossibility theorem (Arrow, 1963), which concerns non-anonymous voters and yields a dictatorship conclusion. We discuss this gap in Section 6.

1.6 Outline

Section 2 sets up the anonymous voting setting as an instance of the tie-breaking framework. Section 3 identifies when symmetric profiles exist. Section 4 states and proves the impossibility. Section 5 applies the characterization and decomposition theorems. Section 6 compares the result to Arrow's theorem.

2 The Anonymous Voting Instantiation

We fix a finite set $\mathcal{N}$ of *alternatives* with $|\mathcal{N}| = m \geq 3$ and a positive integer $n \geq 1$ representing the number of voters. We write $\mathcal{L}(\mathcal{N})$ for the set of linear orders (strict total orders) on $\mathcal{N}$, and $\mathcal{W}(\mathcal{N})$ for the set of weak orders (complete, transitive, and reflexive relations) on $\mathcal{N}$. We write $\succeq_\top$ for the trivial weak order on $\mathcal{N}$ in which all alternatives are tied, i.e., $x \sim y$ for all $x, y \in \mathcal{N}$.

Definition 2.1 (Anonymous Profile)**.** An *anonymous profile of size* n is a function

$$h \colon \mathcal{L}(\mathcal{N}) \to \mathbb{Z}_{\geq 0} \quad \text{with} \quad \textstyle\sum_L h(L) = n.$$

The value $h(L)$ records the number of voters who submitted the ballot L, without tracking which voter submitted which ballot. We write $\mathcal{I}_n$ for the set of anonymous profiles of size n.

The symmetric group $\mathrm{Sym}(\mathcal{N})$ acts on $\mathcal{L}(\mathcal{N})$ by relabeling alternatives: for $\sigma \in \mathrm{Sym}(\mathcal{N})$ and $L \in \mathcal{L}(\mathcal{N})$, the relabeled order $\sigma \cdot L$ ranks $\sigma(x)$ above $\sigma(y)$ whenever L ranks x above y. This action extends to $\mathcal{I}_n$ by setting $(\sigma \cdot h)(L) := h(\sigma^{-1} \cdot L)$ for each $L \in \mathcal{L}(\mathcal{N})$, making $\mathcal{I}_n$ a $\mathrm{Sym}(\mathcal{N})$-set in the sense of Feys (2026, Definition 1).

Definition 2.2 (Neutrality and Resoluteness)**.** Let $f \colon \mathcal{I}_n \to \mathcal{W}(\mathcal{N})$ assign to each anonymous profile a weak order on $\mathcal{N}$.

- The rule f is *neutral* if $f(\sigma \cdot h) = \sigma \cdot f(h)$ for every $\sigma \in \mathrm{Sym}(\mathcal{N})$ and every $h \in \mathcal{I}_n$.
- The rule f is *resolute* if $f(h) \in \mathcal{L}(\mathcal{N})$ for every $h \in \mathcal{I}_n$.

Remark 2.3 (Neutrality Is Anonymity in the Framework)**.** In the language of Feys (2026), neutrality is the anonymity condition of Feys (2026, Definition 7), applied to the information space $\mathcal{I}_n$ with the all-tied standings $\succeq_\top$. Indeed, every weak order on $\mathcal{N}$ trivially refines $\succeq_\top$, so the refinement condition is automatic, and the anonymity condition reduces to $\mathrm{Sym}(\mathcal{N})$-equivariance. We use the term "neutral" rather than "anonymous" to avoid confusion with *voter* anonymity, which is built into the histogram representation and plays no further role.

3 Symmetric Anonymous Profiles

The impossibility theorem of Feys (2026) applies whenever the information space contains a symmetric input. We now identify exactly when $\mathcal{I}_n$ contains such an input, and the answer turns out to be the arithmetic condition that governs the entire ANR tradition.

An input $(h, \succeq)$ is *symmetric* in the sense of Feys (2026, Definition 8) if some nontrivial element of $\mathrm{Stab}(h, \succeq)$ moves an element of $\mathcal{N}$. In our setting the standings are always $\succeq_\top$ (fixed by every permutation), so h is symmetric if and only if $\mathrm{Stab}(h)$ contains a nontrivial element.

Proposition 3.1 (Arithmetic Characterization of Symmetric Profiles)**.** *Let $|\mathcal{N}| = m \geq 3$ and $n \geq 1$. The space $\mathcal{I}_n$ contains a symmetric profile if and only if* $\gcd(n, m!) > 1$.

The proof turns on the single group-theoretic fact that also drives the impossibility theorem itself: no nontrivial permutation of $\mathcal{N}$ can fix a linear order on $\mathcal{N}$.

Proof. ($\Rightarrow$) Suppose $h \in \mathcal{I}_n$ is symmetric, witnessed by a nontrivial $\sigma \in \mathrm{Stab}(h)$. Let p be a prime dividing the order of σ; replacing σ by an appropriate power, we may assume σ has order p. Since $\sigma \in \mathrm{Sym}(\mathcal{N})$ and $|\mathcal{N}| = m$, the order of σ divides $m!$, so $p \mid m!$.

We claim that every $\langle\sigma\rangle$-orbit on $\mathcal{L}(\mathcal{N})$ has size exactly p. Indeed, since σ has prime order p, every $\langle\sigma\rangle$-orbit has size 1 or p. An orbit of size 1 would be a linear order fixed by σ, contradicting the key observation underlying Theorem A of Feys (2026), namely that no nontrivial permutation of $\mathcal{N}$ can fix a linear order. Hence, every orbit has size exactly p.

Since $\sigma \cdot h = h$, the histogram h is constant on each orbit. Writing $c_1, \ldots, c_r$ for the common values on the r orbits, we obtain $n = p \sum_{i=1}^{r} c_i$, so $p \mid n$. Since p divides both n and $m!$, it follows that $\gcd(n, m!) > 1$.

($\Leftarrow$) Suppose $\gcd(n, m!) > 1$, and let p be a prime dividing both n and $m!$. Since $p \mid m!$, there exists $\sigma \in \mathrm{Sym}(\mathcal{N})$ of order p. By the argument above, σ acts on $\mathcal{L}(\mathcal{N})$ with every orbit of size p. Since $p \mid n$, the quotient n/p is a positive integer. Pick any orbit $O = \{L, \sigma \cdot L, \ldots, \sigma^{p-1} \cdot L\}$ and define

$$h(M) := \begin{cases} n/p & \text{if } M \in O, \\ 0 & \text{otherwise.} \end{cases}$$

The total is $p \cdot (n/p) = n$, and σ permutes the elements of O cyclically with equal multiplicities, so $\sigma \cdot h = h$. Hence, h is symmetric. □

Remark 3.2 (The Arithmetic and the Group Theory)**.** Proposition 3.1 connects two apparently different objects: the number-theoretic condition $\gcd(n, m!) > 1$ and the abstract

condition that a symmetric input exists. In fact, both are manifestations of the same group-theoretic fact. The condition $\gcd(n, m!) > 1$ says that some prime p divides both n and $m!$, and the proof shows that this is exactly the condition under which a p-element permutation of the alternatives can act on $\mathcal{L}(\mathcal{N})$ in a way that is compatible with a histogram of total size n. The abstract impossibility theorem of Feys (2026) then does the rest.

Example 3.3 (Three Instances)**.** We illustrate the proposition.

The Condorcet histogram ($n = 3$, $m = 3$). Take $\mathcal{N} = \{a, b, c\}$ and the three cyclic ballots $a \succ b \succ c$, $b \succ c \succ a$, $c \succ a \succ b$, each with multiplicity 1. The cyclic permutation $\rho = (a\ b\ c)$ permutes these three ballots cyclically and preserves multiplicities, so $\rho \in \mathrm{Stab}(h_C)$. This is consistent with $\gcd(3, 6) = 3 > 1$.

A two-type histogram ($n = 2$, $m = 3$). Take one ballot of type $a \succ b \succ c$ and one of type $b \succ a \succ c$. The transposition $(a\ b)$ swaps the two types with equal multiplicities, so $(a\ b) \in \mathrm{Stab}(h)$. This is consistent with $\gcd(2, 6) = 2 > 1$.

A non-symmetric case ($n = 5$, $m = 3$). We have $\gcd(5, 6) = 1$, so by Proposition 3.1, no symmetric profile of size 5 on three alternatives exists. Indeed, any nontrivial element of $\mathrm{Sym}(\{a, b, c\})$ has order 2 or 3, and neither 2 nor 3 divides 5. Observe that Moulin's existence result guarantees, for exactly this parameter pair, that a neutral resolute rule *does* exist.

4 The Impossibility as a Corollary

We now state and prove the main result. The theorem is not new; it is the impossibility direction of the ANR theorem for ranking output, established by Bubboloni and Gori (2014). What is new is the proof: it is a direct application of Theorem A of Feys (2026), a framework-level result with no reference to voting.

Theorem 4.1 (The ANR Impossibility for Rankings)**.** *Let $|\mathcal{N}| = m \geq 3$ and $n \geq 1$ with $\gcd(n, m!) > 1$. There is no neutral resolute function $f \colon \mathcal{I}_n \to \mathcal{L}(\mathcal{N})$.*

Proof. By Proposition 3.1, $\mathcal{I}_n$ contains a symmetric profile. A neutral resolute function $f \colon \mathcal{I}_n \to \mathcal{L}(\mathcal{N})$ is an anonymous strict tie-breaking rule in the sense of Feys (2026, Definition 6) on the information space $\mathcal{I}_n$ with standings $\succeq_\top$. By Theorem A of Feys (2026), no such rule exists. □

The geometric picture is the following. A symmetric profile h_* is a ρ-fixed point in $\mathcal{I}_n$ for some nontrivial $\rho \in \mathrm{Sym}(\mathcal{N})$. Neutrality requires $f(h_*) = \rho \cdot f(h_*)$, so $f(h_*)$ must itself be a ρ-fixed point. But the codomain is $\mathcal{L}(\mathcal{N})$, and no linear order is fixed by a nontrivial permutation. An equivariant map must send a fixed point to a fixed point, and there is none in the codomain. Figure 1 illustrates this for the Condorcet histogram.

Remark 4.2 (What the Proof Reveals)**.** The classical proofs of the ANR impossibility (Moulin, 1983, Bubboloni and Gori, 2014) operate within the voting-specific framework of profiles, voter permutations, and alternative permutations. The proof above operates at a different level: it treats anonymous profiles as instances of an abstract $\mathrm{Sym}(\mathcal{N})$-set and applies a theorem that was proved without reference to voting. The broader lesson is that

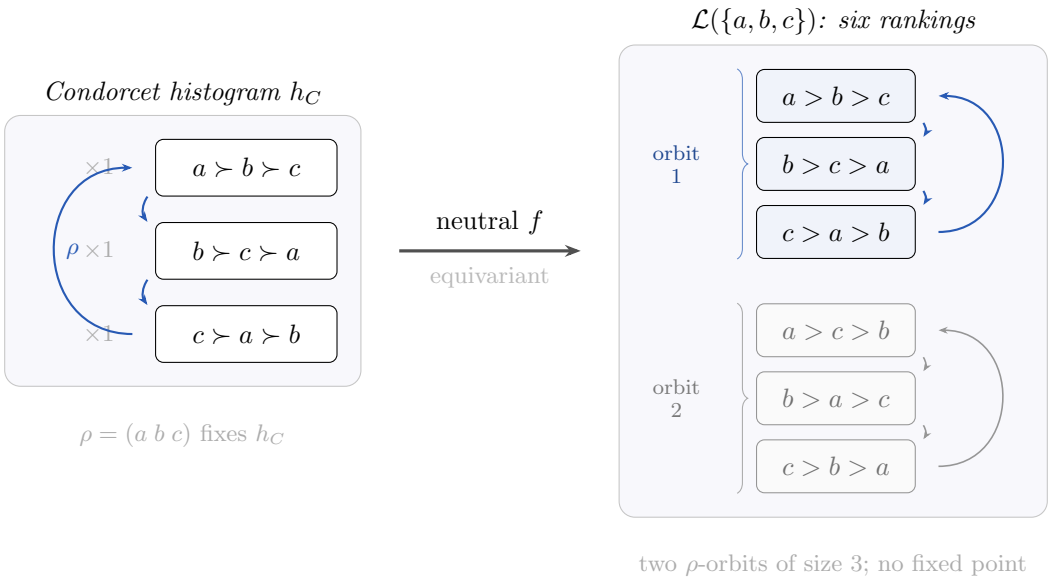


Figure 1: The impossibility for the Condorcet histogram. The cyclic permutation $\rho = (a\ b\ c)$ fixes h_C (left). Neutrality requires $f(h_C) = \rho \cdot f(h_C)$, so $f(h_C)$ must be a ρ-fixed point in $\mathcal{L}(\{a, b, c\})$. The six linear orders split into two ρ-orbits of size 3 (right), neither of which contains a fixed point. An equivariant map must send a fixed point to a fixed point, and the codomain has none. This is the obstruction of Theorem A of Feys (2026), instantiated for anonymous voting.

the ANR impossibility has the same source as the impossibility of strict tie-breaking in chess tournaments, in round-robin sports leagues, in cooperative games with symmetric players, and in network centrality on graphs with nontrivial automorphism groups. Theorem A proves the obstruction once, in the abstract. The present note, and each of these other settings, are instances.

Corollary 4.3 (Unconditional Version)**.** *Let $|\mathcal{N}| = m \geq 3$ and $n \geq 2$. If $\mathcal{I} = \bigcup_{k=1}^{n} \mathcal{I}_k$ is the space of all anonymous profiles of size at most n, then no neutral resolute function $f \colon \mathcal{I} \to \mathcal{L}(\mathcal{N})$ exists.*

Proof. Since $m \geq 3$, we have $2 \mid m!$, and hence $\gcd(2, m!) \geq 2 > 1$. The subspace $\mathcal{I}_2$ is $\mathrm{Sym}(\mathcal{N})$-invariant, so the restriction of f to $\mathcal{I}_2$ is a neutral resolute function $\mathcal{I}_2 \to \mathcal{L}(\mathcal{N})$. By Proposition 3.1, $\mathcal{I}_2$ contains a symmetric profile, and the result follows from Theorem 4.1. □

5 The Orbit Partition on Anonymous Profiles

When the output is not required to be a strict linear order, the impossibility dissolves. The orbit partition Ω provides the canonical replacement: a partition-valued rule exists uniquely, and every reasonable strict extension is the canonical partition followed by a completion. We now describe how this picture plays out on anonymous profiles.

5.1 Forced Ties

For any anonymous profile h, the orbit partition $\Omega(h) := \Omega(h, \succeq_\top)$ is the partition of $\mathcal{N}$ whose blocks are the orbits of $\mathrm{Stab}(h)$ acting on $\mathcal{N}$. Two alternatives x, y lie in the same block if and only if some $\sigma \in \mathrm{Stab}(h)$ satisfies $\sigma(x) = y$.

Proposition 5.1 (Forced Ties)**.** *Let $f\colon \mathcal{I}_n \to \mathcal{W}(\mathcal{N})$ be neutral. For every $h \in \mathcal{I}_n$ and every $\sigma \in \mathrm{Stab}(h)$, the weak order $f(h)$ is fixed by σ. In particular, if $\sigma(x) = y$ for distinct $x, y \in \mathcal{N}$, then $x \sim_{f(h)} y$.*

Proof. Neutrality gives $f(\sigma \cdot h) = \sigma \cdot f(h)$, and $\sigma \in \mathrm{Stab}(h)$ gives $\sigma \cdot h = h$, so $f(h) = \sigma \cdot f(h)$. Suppose $x \succ_{f(h)} y$. Since $f(h) = \sigma \cdot f(h)$, we have $\sigma(x) \succ_{f(h)} \sigma(y)$. Following the orbit $x, \sigma(x), \sigma^2(x), \ldots$ and applying transitivity yields $x \succ_{f(h)} x$, a contradiction. The same argument with the roles of x and y interchanged rules out $y \succ_{f(h)} x$. Hence, $x \sim_{f(h)} y$. □

The conceptual reading is as follows. Alternatives in the same block of $\Omega(h)$ are intrinsically indistinguishable from the histogram's standpoint: some symmetry of h exchanges them. No neutral rule can rank one above the other, because the rule has no honest basis for doing so.

Example 5.2 (Three Histograms, Three Orbit Partitions)**.** We compute $\Omega(h)$ for three representative histograms on $\mathcal{N} = \{a, b, c\}$, illustrating the range from maximal to minimal forced ties.

Maximal symmetry: the Condorcet histogram. The cyclic permutation $\rho = (a\ b\ c)$ lies in $\mathrm{Stab}(h_C)$ and acts transitively on $\{a, b, c\}$. Hence, $\Omega(h_C) = \{\{a, b, c\}\}$, a single block. Any neutral rule must output $a \sim b \sim c$.

Partial symmetry: two balanced types. Consider ballot types $L_1 = (a \succ b \succ c)$ and $L_2 = (b \succ a \succ c)$, each with multiplicity $k \geq 1$. The transposition $(a\ b)$ maps L_1 to L_2 and vice versa, so $(a\ b) \in \mathrm{Stab}(h)$. One checks that no other nontrivial permutation fixes h. Hence, $\Omega(h) = \{\{a, b\}, \{c\}\}$. Any neutral rule must tie a and b, but is free to rank c above or below them.

No symmetry: a generic histogram. Let h assign multiplicity n to a single ballot $a \succ b \succ c$ and 0 to all others. No nontrivial permutation fixes a linear order, so $\mathrm{Stab}(h) = \{\mathrm{id}\}$ and $\Omega(h)$ is discrete. No ties are forced.

5.2 The Characterization and the Decomposition

The characterization theorem (Theorem B of Feys 2026) identifies Ω as the unique partition-valued rule satisfying two axioms: symmetry-saturation (orbit-related alternatives must be grouped together) and maximal-fineness (alternatives not related by any symmetry must be separated). In the language of voting, Ω is the unique neutral rule that ties exactly the alternatives the profile forces to be tied, and no others. The partition is not too coarse and not too fine. It sits at the unique level where distinguishability matches the input's information content.

By the decomposition theorem (Theorem C of Feys 2026), every partition-consistent strict rule factors as Ω followed by a completion. A completion consists of a linear order within each orbit block and an ordering of the blocks. The orbit partition is the *forced* part

of any honest aggregation. It is not chosen, calibrated, or assumed, but determined by the profile's symmetry structure. The completion is the *free* part, the irreducibly arbitrary data that must be supplied externally.

This is the content of the slogan from Feys (2026): *tie-breaking systems are honest until forced to be arbitrary.* On anonymous profiles, the orbit partition records exactly the boundary between the honest and the arbitrary.

6 Comparison with Arrow's Theorem

We now compare Theorem 4.1 with Arrow's impossibility theorem (Arrow, 1963). The two results share a common geometric root but differ in axioms, scope, and conclusions.

Axioms. Arrow's theorem assumes four properties of a social welfare function on non-anonymous profiles: universal domain, the Pareto principle, independence of irrelevant alternatives (IIA), and non-dictatorship. Theorem 4.1 assumes two properties on anonymous profiles: neutrality and resoluteness. The axiom sets are largely disjoint. Arrow assumes Pareto and IIA but not neutrality; we assume neutrality but not Pareto or IIA.

Voter identity. Arrow's theorem concerns non-anonymous profiles: each voter has a name, and the conclusion identifies a specific voter (the dictator). Our result concerns anonymous profiles: voters have no names, and the conclusion is a non-existence statement.

Output. Arrow's theorem allows weak-order output and still obtains an impossibility in the form of dictatorship. Theorem 4.1 requires resolute output. When ties are allowed in our setting, the impossibility dissolves and is replaced by the characterization of Ω (Section 5).

The shared obstruction. Despite these differences, both results rest on the same group-theoretic fact: a nontrivial permutation cannot fix a linear order on $\mathcal{N}$. In our setting, this fact is applied directly via Theorem A of Feys (2026). In Arrow's setting, the same fact operates indirectly: the classical proof machinery (contagion, ultrafilter arguments, Pareto) propagates a local obstruction into a global conclusion about voter coalitions.

The fundamental obstacle to proving full Arrow. The framework's symmetry group $\mathrm{Sym}(\mathcal{N})$ acts on *alternatives*. Arrow's dictatorship conclusion is a statement about *voters*. No theorem about $\mathrm{Sym}(\mathcal{N})$-equivariance can identify a specific voter, because $\mathrm{Sym}(\mathcal{N})$ does not act on the voter set. Furthermore, in the non-anonymous information space $\mathcal{L}(\mathcal{N})^V$, where V denotes the set of voters, with $\mathrm{Sym}(\mathcal{N})$ acting by relabeling alternatives, the stabilizer $\mathrm{Stab}(\pi)$ is trivial for every profile π, since any $\sigma \in \mathrm{Stab}(\pi)$ must fix every voter's ballot and no nontrivial permutation fixes any linear order. There are no symmetric inputs, and Theorem A cannot apply. This is not a gap in cleverness; it is a category mismatch. Extending the framework to prove the full Arrow theorem would require a second symmetry group, $\mathrm{Sym}(V)$ acting on voters, and voter-side analogues of the orbit partition. We view this as a genuine open problem.

7 Conclusion

We showed that the ANR impossibility for rankings (no neutral resolute rule exists when $\gcd(n, m!) > 1$) follows as a direct corollary of Theorem A, the impossibility theorem of the abstract tie-breaking framework of Feys (2026). The proof uses no voting-specific machinery: no decisive coalitions, no profile constructions beyond a single symmetric histogram, no Pareto principle.

The conceptual point is that the ANR impossibility is driven by the same obstruction that makes strict tie-breaking impossible in chess, in sports, in cooperative games, and in network centrality: a nontrivial permutation cannot fix a linear order. The tie-breaking framework proves this obstruction once, in the abstract, and the ANR impossibility follows as one of its instances.

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